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Launch Years initiative

# Transition to College Mathematics Course Framework

version 2.0


Launch Years
*an initiative of*

The University of Texas at Austin
Charles A. Dana Center

CCRC COMMUNITY COLLEGE RESEARCH CENTER
TEACHERS COLLEGE, COLUMBIA UNIVERSITY

Achieve

ESG Education Strategy Group

Association of Public & Land-Grant Universities

## About the Dana Center

The Charles A. Dana Center develops and scales mathematics and science education innovations to support educators, administrators, and policymakers in creating seamless transitions throughout the K–16 system for all students, especially those who have historically been underserved. We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement.

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations.

## About Launch Years

Launch Years is an initiative led by the Charles A. Dana Center at The University of Texas at Austin—in collaboration with Community College Research Center, Achieve, Education Strategy Group, and the Association of Public and Land-grant Universities—focused on addressing systemic barriers that prevent students from succeeding in mathematics and progressing to postsecondary and career success. Leveraging work within states, the initiative seeks to modernize math in high school through relevant and rigorous math courses as well as through policies and practices leading to more equitable outcomes for all students. Learn more at: **utdanacenter.org/launch-years**.

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Over the last 20 years, mathematics has become increasingly important to a growing number of fields of study and their related professions. In 1998, the National Science Foundation released "International Assessment of the U.S. Mathematical Sciences" (Odom Report), which listed 11 fields of study that interface with mathematics, including physics, chemistry, economics, and manufacturing. The National Research Council's 2013 report, "The Mathematical Sciences in 2025," expanded this number to 21 and predicted continued growth. The new fields of study added to the list were notable—entertainment, social networks, ecology, computer science, information processing, marketing, and defense.

Unfortunately, the current system of mathematics education fails to meet the needs of many of our students. **This is unacceptable**. It is reprehensible that so many students' opportunities to succeed are limited by their race or economic class. We have a moral and a professional obligation to create the conditions necessary for every student to succeed.

The Launch Years initiative intends to create those conditions through two overarching aims.

The first is to improve learning opportunities for each student during the last two years of high school and into the transition to their postsecondary education and other future endeavors.

The second is to dismantle institutional and systemic barriers that block equitable access and opportunities to succeed in mathematics, especially for students who are Black, Latinx, or Native American, or who come from low-income communities.

The Launch Years vision is to build, scale, and sustain policies, practices, and structures that ensure that each student has equal access to, and successfully engages in:

- **mathematics courses** with rigorous, relevant, engaging, high-quality, and inclusive instruction that is responsive to the needs of individual students and that is informed by multiple measures of achievement that are economically and culturally inclusive;
- **mathematics pathways** that are well articulated from high school to and through postsecondary education and careers, that are personally and socially relevant, and that enable students to move across pathways as their interests and aspirations evolve; and
- **individualized academic, career, and other student supports** that respect and promote student and family decision-making and that enable students to explore options, make strategic choices, and set and achieve informed goals.

To support the first aim of the Launch Years initiative—to improve learning opportunities for each student during the last two years of high school and into the transition to postsecondary education—the Charles A. Dana Center at The University of Texas at Austin has collaborated with stakeholders from K–12 and higher education to develop a course framework for a senior-level transition mathematics course.

A design team, comprising content experts in K–12 and higher education, worked together to develop the course framework and sought input from experts in a variety of fields to inform the design team's work. The framework contained in this document describes a course that encompasses multiple pathways and also supports students' social, emotional, and academic development—an often-overlooked aspect of education that research indicates is crucial to students' ability to thrive in school, career, and life.

We recognize that simply implementing new courses is not enough. We commit to challenging and eliminating institutional and systemic barriers to students' opportunities to access—and succeed in—mathematics. This commitment includes proactively working with partners to change institutional cultures and educator mindsets toward recognizing and building upon student assets and student strengths.

# Acknowledgments

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# Introduction and Purpose

This year-long transition course is intended for 12th grade students whose academic performance prior to their senior year or performance on an accepted external college readiness measure indicates that they are not yet ready to perform entry-level college mathematics coursework. The goal of the course is to ensure that these students meet a college readiness measure by the end of their senior year and are prepared for most entry-level, credit-bearing college mathematics courses, especially quantitative reasoning, statistics, or college algebra.

# Content Overview

The transition course covers a variety of mathematical and statistical topics needed to prepare students for success in multiple mathematics pathways. It is expected that students enrolled in this course will have successfully completed Algebra I/Integrated Mathematics I (and any associated EOC exam) and Geometry/Integrated Mathematics II. In addition, students should have taken a third year of mathematics, such as statistics or Algebra II (or its equivalent).

In this course, students will connect and use multiple strands of mathematics to solve problems. Topics include:

- Numeracy, including facility in representing large and small numbers, fluency with arithmetic operations, and facility with estimation skills;
- Proportional reasoning in situations involving rates, ratios, percentages, and measurement;
- Manipulation and evaluation of expressions and formulas that model physical and geometric phenomena;
- Key characteristics of linear, quadratic, exponential, simple rational, and monomial functions through verbal, numerical, algebraic, and graphical interpretations;
- Linear functions and related equations, inequalities, and systems;
- Quadratic and exponential functions and related equations;
- Univariate and categorical data representation and analysis;
- Bivariate data representation and analysis;
- Probabilistic reasoning in situations involving chance or risk; and
- Evaluation of reports from statistical studies.

The transition course supports students' social, emotional, and academic development (SEAD) through explicit instruction about and development of attitudes, skills, and strategies needed to succeed in college coursework. These include:

- A growth mindset that includes an understanding that math ability can change over time through effective effort and strategies;
- Perceiving and valuing mathematical understanding as personally relevant;
- Effective collaboration;
- Building and utilizing a support network;
- Seeking help and acting on feedback;

- Self-regulation;
- Persisting through challenging tasks; and
- Setting and monitoring goals.

In addition, the course explicitly develops students' ability to engage in important mathematical practices and processes to develop and demonstrate their mathematical understanding. These include:

- Selecting and using technology (e.g., calculator, spreadsheet, computer algebra package, statistical package, dynamic geometry software) appropriate for a particular context or purpose;
- Interpreting and communicating quantitative information and mathematical and statistical concepts using language and representations appropriate to the context and intended audience;
- Making sense of problems, developing strategies to find solutions, and persevering in solving them; and
- Reasoning, modeling, and making decisions with given information, including understanding and critiquing the arguments of others.

These practices and processes describe ways in which students are expected to engage with the content. Teachers should ensure student engagement through the activities they provide and through facilitation of classroom instruction.

# Course Design Principles

Curricular materials and classroom instruction for this course should engage students in meaningful interactions that amplify the learning through social interaction; facilitate transfer of math and SEAD skills; and create an inclusive learning context for all learners, particularly for students who feel disconnected from mathematics and disaffected by the learning process.

The following design principles describe how curricular materials and classroom instruction for the transition course should be structured to support a coherent and engaging experience. Developers should use these standards to create curricular materials that are true to the vision of the course, and educators should also use the design principles when building a repertoire of pedagogical strategies for use in teaching the course.

We are aware that many students and teachers already engage in these behaviors. Our hope is that these design principles will be seen as reinforcing and supportive. The spirit of this framework recognizes that, at some levels, we are all learners, and are growing in our understanding of mathematics, one another, and the world around us.

## Transition Course Design Principles

| Design Principle | Students will . . . | Teachers will . . . |
|---|---|---|
| **Active Learning.** The course provides regular opportunities for students to actively engage in discussions and tasks using a variety of different instructional strategies (e.g., hands-on and technology-based activities, small group collaborative work, facilitated student discourse, interactive lectures). | • Be active and engaged participants in discussion, in working on tasks with classmates, and in making decisions about the direction of instruction based on their work.<br>• Actively support one another's learning.<br>• Discuss course assignments and concepts with the instructor and/or classmates outside of class. | • Provide activities and tasks with accessible entry points that present meaningful opportunities for student exploration and co-creation of mathematical understanding.<br>• Facilitate students' active learning of mathematics and statistics through a variety of instructional strategies, including inquiry, problem solving, critical thinking, and reflection, with limited time spent in "direct teach" activities.<br>• Create a safe, student-driven classroom environment in which all students feel a sense of belonging to the class and the discipline, are not afraid to take risks or make mistakes, and are able to make decisions about the direction for instruction through the results of their exploration of mathematics and statistics. |

## Transition Course Design Principles

| Design Principle | Students will . . . | Teachers will . . . |
| --- | --- | --- |
| **Constructive Perseverance.** The course supports students in developing the tenacity, persistence, and perseverance necessary for learning mathematics and statistics, for using mathematics and statistics to tackle authentic problems, and for being successful in post-high school endeavors. | • Make sense of tasks by drawing on and making connections with their prior understanding and ideas.<br>• Persevere in solving problems and realize that it is acceptable to say, "I don't know how to proceed here," but that it is not acceptable to give up; seek help from appropriate sources to continue to move forward.<br>• Help one another by sharing strategies and solution paths rather than simply giving answers.<br>• Reflect on mistakes and misconceptions to improve their mathematical understanding.<br>• Seek to understand and address the reasons for their struggles to help them make progress in solving problems and overcoming challenges in the course.<br>• Understand that, while they may struggle at times with mathematics tasks, breakthroughs often emerge from confusion and struggle. | • Provide instruction and information about the role of productive struggle in learning.<br>• Pose tasks on a regular basis that require a high level of cognitive demand.<br>• Allow students to engage in productive struggle with challenging tasks.<br>• Anticipate what students might struggle with during a lesson and be prepared to support them productively through the struggle.<br>• Give students time to struggle with tasks and ask questions that scaffold students' thinking without stepping in to do the work for them.<br>• Praise students for their effective efforts in making sense of mathematical ideas and for their perseverance in reasoning through problems and in overcoming setbacks and challenges in the course.<br>• Help students realize that confusion and errors are a natural part of learning by facilitating discussions on mistakes, misconceptions, and struggles.<br>• Provide students with low-stakes opportunities to fail and learn from failure.<br>• Provide regular opportunities for students to self-monitor, evaluate, and reflect on their learning, both individually and with their peers. |

## Transition Course Design Principles

| Design Principle | Students will . . . | Teachers will . . . |
|---|---|---|
| **Problem Solving.** The course provides opportunities for students to make sense of problems and persist in solving them. | • Apply previously learned strategies to solve unfamiliar problems.<br>• Explore and use multiple solution methods.<br>• Share and discuss different solution methods.<br>• Be willing to make and learn from mistakes in the problem-solving process.<br>• Use tools and representations, as needed, to support their thinking and problem solving. | • Present tasks that require students to find or develop a solution method.<br>• Provide tasks that allow for multiple strategies and solution methods, including transfer of previously developed skills and strategies to new contexts.<br>• Provide opportunities to share and discuss different solution methods.<br>• Model the problem-solving process using various strategies.<br>• Encourage and support students to explore and use a variety of approaches and strategies to make sense of and solve problems. |
| **Authenticity.** The course presents mathematics and statistics as necessary tools to model and solve problems that arise in the real world. | • Recognize specific ways in which mathematics is used in everyday decision making.<br>• Recognize problems that arise in the real world that can be solved with mathematics or statistics.<br>• Contribute meaningful questions that can be answered using mathematics. | • Provide opportunities to solve problems that are relevant to students, both in class and on assessments, that utilize real-world—not contrived—contexts.<br>• Provide opportunities for students to pose questions that can be answered using mathematics or statistics and answer them. |
| **Context and Interdisciplinary Connections.** The course presents mathematics and statistics in context and connects mathematics and statistics to various disciplines and everyday experiences. | • Contribute personal experiences, where appropriate, that connect to classroom experiences.<br>• Actively seek connections between classroom experiences and the world outside of class. | • Provide opportunities for students to share their personal backgrounds and interests, including cultural values, and help make the connection between what is important in students' lives and future aspirations, and what they are learning in mathematics.<br>• Provide activities and tasks that use real data, whenever possible.<br>• Provide activities and tasks that illustrate authentic applications.<br>• Provide activities and tasks that explore problems from a variety of academic disciplines, programs of study, and careers, and that are culturally relevant. |

## Transition Course Design Principles

| Design Principle | Students will . . . | Teachers will . . . |
|---|---|---|
| **Communication.** The course develops students' ability to communicate about and with mathematics and statistics in contextual situations. | • Present and explain ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse using discipline-specific terminology, language constructs, and symbols.<br>• Seek to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others.<br>• Listen carefully to and critique the reasoning of peers using examples to support or counterexamples to refute arguments. | • Introduce concepts in a way that connects students' experiences to course content and that bridges from informal contextual descriptions to formal definitions.<br>• Clarify the use of mathematical and statistical terminology and symbols, especially those used in different contexts or different disciplines.<br>• Engage students in purposeful sharing of mathematical ideas, reasoning, and approaches using varied representations.<br>• Support students in developing active listening skills and in asking clarifying questions to their peers in a respectful manner that deepen understanding.<br>• Facilitate discourse by positioning students as authors of ideas who explain and defend their approaches.<br>• Provide regular opportunities for students to write about mathematics and statistics with tasks to deepen understanding and with authentic contextual tasks that require use of mathematical or statistical concepts (e.g., writing a brief paper that interprets the results of a statistical study).<br>• Scaffold instruction to support students in developing the required reading and writing skills. |

## Transition Course Design Principles

| Design Principle | Students will . . . | Teachers will . . . |
|---|---|---|
| **Technology.** The course leverages technology to develop conceptual understanding and to facilitate active learning by enabling students to directly engage with and use mathematical concepts. | • Use technology to assist them in visualizing and understanding important mathematical concepts and as a support to problem solving.<br>• Allow technology to assist investigations with problems that might otherwise be too difficult or time-consuming to explore.<br>• Consider the relative usefulness of a range of tools in particular contexts.<br>• Understand that the use of tools or technology does not replace the need for an understanding of reasonableness of results or how the results apply to a given context. | • Use technology to assist students in visualizing and understanding important mathematical concepts and support students' mathematical reasoning and problem solving.<br>• Leverage technology as a tool that can expand the scope of mathematical ideas and problems that students can investigate.<br>• Support students in using technology for more than just answer-getting and in making appropriate choices of technology to use, depending on the problem to be solved.<br>• Be mindful of effective uses of technology and plan carefully for strategic use of technology. |

# Sample Student Learning Outcomes

The following SEAD and mathematics outcomes provide more detail about the content described in the Content Overview section above. In order for students to acquire the knowledge and skills outlined in the SEAD outcomes, course materials should provide explicit instruction for each outcome, combined with opportunities for students to apply what they have learned as they engage with the mathematics content.

To acquire the knowledge and skills outlined in the mathematics outcomes, students should, whenever possible, engage in learning motivated by an authentic context and apply their knowledge and skills to solve real-world problems appropriate for and of interest to seniors in high school. The italicized text that follows some outcomes provides additional clarification and, in some cases, illustrative examples.

## Social, Emotional, and Academic Development

Students should develop and strengthen social-emotional skills and competencies critical to academic success, including competencies in the cognitive, social and interpersonal, and emotional domains.

Students will:

**SEAD.1** Recognize situations for which collaboration is an effective strategy, identify features of effective and productive collaborative work groups, and develop strategies for overcoming group work challenges.

**SEAD.2** Learn that intelligence is malleable and understand how purposeful engagement, persistence, and intelligence are related.

**SEAD.3** Engage in productive academic behaviors, including help-seeking, self-regulation, and utilizing feedback.

**SEAD.3a** Recognize when help is needed with a task, identify sources of help, and develop and apply a variety of strategies for seeking help.

**SEAD.3b** Monitor and adjust attitudes, emotions, and thoughts when facing challenging tasks or academic setbacks.

**SEAD.3c** Actively seek and listen to feedback and act on feedback to improve performance.

**SEAD.4** Maintain motivation and persistence through a variety of strategies, including identifying and adjusting habits and beliefs that have interfered with success; applying metacognitive awareness to plan, monitor, evaluate, and reflect on learning; and setting and monitoring goals.

## Numeric Reasoning

Students should solve authentic problems in a variety of contexts that require number sense and the ability to apply concepts of numeracy to investigate and describe quantitative relationships.

Students will:

**NR.1** Engage in problem solving that demonstrates an understanding of real numbers, including notation and operations.

**NR.1a** Recognize subsets of real numbers and the notation used to describe the subsets, including the roster method, set builder notation, and interval notation, used in contextual settings. Use a Venn diagram to describe the relationship between subsets of real numbers. Know when different sets of numbers are appropriate to use.

*For example: Recognize why the natural numbers are useful for describing terms in a sequence to model the concentration of a drug present in the blood stream at set time intervals after the initial dose. Choose and use set notation and symbols as appropriate for subsets of numbers, such as domains and ranges of functions, and sample spaces and events for chance experiments.*

**NR.1.b** Engage in problem solving that demonstrates fluency with arithmetic operations on rational numbers. Use precise mathematical language when communicating about rational numbers.

*For example: Predict the effects of multiplying any real number by a rational number between 0 and 1. Represent real numbers on a number line. Use the order of operations to simplify expressions including exponents and to solve problems (e.g., to identify errors in formulas in a spreadsheet).*

**NR.1.c** Engage in problem solving that uses appropriate notation for radicals in terms of rational exponents. Reason quantitatively to emphasize number sense and reasonableness when estimating the value of a radical and when distinguishing between exact and approximate values.

*For example: Approximate the value of an irrational number like* $\sqrt{52}$ *or* $\sqrt[3]{10}$ *using perfect squares or perfect cubes, and locate the rational number approximation of the radical on a number line. Know that the exact value of a distance found by the distance formula may be left in radical form, but an approximation of distance may be more useful in context.*

**NR.1.d** Represent rational numbers in equivalent forms using fractions, decimals, and percentages. Compare the size of numbers in different forms and interpret the meaning of numbers in different forms.

*For example: Interpret the meaning of percentages greater than 100% and justify whether such a percentage is possible in a given context. Solve authentic problems involving numbers in different forms, such as comparing growth of a population expressed as a fraction versus as a percentage.*

**NR.1.e** Solve authentic problems involving calculations with percentages and interpret the results.

*For example: Calculate and understand the impact of a percentage increase or decrease in a contextual situation, such as the difference between a discount of 30% and two consecutive discounts of 15%. Calculate absolute change and explain how it differs from relative change. Solve problems related to personal finance, such as calculating the interest paid on credit card debt in which the rate is based on a credit score; explaining how the length of the pay-off period affects the total interest paid; or demonstrating the relationship between a percentage rate and the amount of interest paid.*

**NR.1.f** Demonstrate an understanding of large and small numbers by interpreting and communicating with different forms (including words, fractions, decimals, standard notation, and scientific notation) and compare magnitudes in context, using inequality symbols appropriately.

*For example: Compare large numbers in context, such as the population of the U.S. compared to the population of the world, using appropriate representations such as scientific notation. Calculate ratios with large numbers such as water use per capita for a large population. Interpret a growth rate less than 1%.*

**NR.2** Use numbers and units appropriately to model and solve real-world problems.

**NR.2.a** Use estimation skills to solve authentic problems, and know why, how, and when to estimate values. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

*For example: Decide when and how to estimate costs in a context and when exact values are necessary. Estimate the number of seats in a large auditorium by counting one row and multiplying by the number of rows. Estimate the number of people in attendance at a large concert by counting the number of people in a random square unit and estimating the area of the crowd.*

**NR.2.b** Identify, use, and record appropriate units of measure within data displays, on graphs, and when solving contextual geometric problems. Use units as a way to understand formulas and problems and to guide the solution of multi-step problems.

*For example: Identify the appropriate units for perimeter, area, and volume, such as when calculating the amount of paint needed to paint a non-rectangular surface. Choose and interpret units consistently in formulas, such as when calculating interest or when determining a time given a constant velocity and distance. Choose and interpret the scale and the origin in graphs and data displays. Solve measurement problems that require using the Pythagorean Theorem, attending to appropriate use of units.*

**NR.2.c** Find and use quantitative information to explore the impact of policies or behaviors, including those with social, economic, or environmental implications, on a population.

*For example: Assess the effects of a small decrease in individual water use on the amount needed by a large population over time. Determine if the minimum wage has kept pace with inflation over time.*

## Proportional Reasoning

Students should represent and solve authentic problems using proportional reasoning with ratios, rates, proportions, and scaling. They should be able to strategically and flexibly utilize various representations to describe, make sense of, and draw conclusions in situations involving proportional reasoning.

Students will:

**PR.1** Solve real-world problems involving ratios and rates, using a variety of representations (including ratio tables, double number lines, percentages, fractions, and decimals).

**PR.1.a** Use rate reasoning to convert between units of measurement in authentic situations.

*For example: Use double number lines to convert between currencies. Use ratio tables to solve problems involving dosages of medicine. Relate rate reasoning to dimensional analysis, and know why and how the process works and when to use it.*

**PR.1.b** Use rate reasoning to solve problems involving unit rates, such as those related to pricing and speed.

**PR.1.c** Use ratio and rate reasoning to explore policies or behaviors, including those with social, economic, or environmental implications, in a population.

*For example: Use individual water-use rates to predict the water used by a population. Use the Consumer Price Index to compare prices over time. Interpret a percentage as a number out of 1,000 (as is common in medical research). Compare risks expressed in ratios with unequal denominators (e.g., 1 in 8 people will have side effects versus 3 in 14). If one YouTube video has approximately 5 dislikes for every 300 likes, and the like-to-dislike ratio for a second video is 400 to 7, determine which video is better liked by generating various representations.*

**PR.2** Analyze, represent, and solve real-world problems involving proportional relationships, with attention to appropriate use of units.

**PR.2.a** Use various representations to determine whether a proportional relationship exists between two quantities based on how the change in one value is associated with the change in the other.

**PR.2.b** Analyze when scaling and shrinking lead to proportional and non-proportional results (e.g., the impact of changing various dimensions on perimeter, area, and volume) and determine whether two figures are similar.

*For example: Determine whether an 8 × 10 inch photo is similar to a 5 × 7 inch photo, using a ratio table or a double number line.*

**PR.2.c** Use proportional reasoning to solve authentic, indirect measurement problems.

*For example: Use a scale to calculate measurements in a graphic or diagram. Apply the 1:12 Americans with Disabilities Act (ADA) standard to design a wheelchair ramp for a 28-inch change in elevation.*

## Statistical and Probabilistic Reasoning

Students should use the language and tools of probability and statistics to quantify uncertainty in a variety of real-world contexts. They should make informed, evidence-based decisions and justify conclusions about populations based on a random sample from that population. They should be able to critically evaluate statements that appear in the popular media involving risk and arguments based on probability.

Students will:

**SR. 1** Summarize, represent, and interpret univariate and categorical data, with and without technology, to describe and compare distributions.

**SR.1.a** Create appropriate graphical representations for univariate data, including dot plots, histograms, box plots, and stem-and-leaf plots. Analyze the shape of the graph to determine which measure of center (mean, median, mode) and variability (interquartile range, mean absolute deviation, standard deviation) is the best choice for describing center and variability.

**SR.1.b** Create and interpret appropriate numerical summaries for center (mean, median, mode) and variability (interquartile range, mean absolute deviation, standard deviation) for univariate data, accounting for possible effects of extreme data points (outliers).

**SR.1.c** Use measures of center and variability appropriate to the shape of the data distribution to compare two or more different data sets to infer possible differences in the populations from which the data were drawn.

**SR.1.d** Summarize categorical data in a two-way frequency table and recognize possible associations and trends in the data. Choose the appropriate direction of conditioning for a given context and calculate the applicable marginal, joint, and conditional relative frequencies. (**Note:** It is not expected that students name or define these specific terms, but rather that they can determine the indicated types of frequencies as appropriate based on context.)

> *For example: From a two-way frequency table summarizing test outcomes for 500 people, some of whom have cancer and some who do not, determine the number of people with cancer given a positive test result and the number of people with a positive test result given that they have cancer. Choose the relative frequency that is the most informative for a given purpose.*

**SR.2** Summarize, represent, and interpret bivariate data to investigate relationships and make predictions.

**SR.2.a** For linear and exponential models, represent paired quantitative data on a scatterplot, use technology to fit a model to the data (using function transformations or regression), and interpret the model in the context of the data. Use the model to make predictions, where appropriate, evaluate the reasonableness of the prediction, and discuss any limitations of the model.

**SR.2.b** For linear models, use technology to compute the correlation coefficient "r", interpret the value of the correlation coefficient, and relate it to the strength and direction of the relationship displayed in a scatterplot. Calculate and interpret the vertical distance between a predicted $y$-value and an observed $y$-value from the data (a residual).

**SR.2.c** Distinguish between correlation and causation.

**SR.3** Analyze statements of chance, risk, and probability that appear in everyday media (including terms such as "unlikely," "rare," or "impossible"). Determine and interpret probabilities of events.

*For example: Interpret statements such as "For a certain population, the lifetime risk of a particular disease is 0.005." Compare incidences of side effects in unequal group sizes. Identify inappropriate risk statements, such as when the size of reference groups is unknown (e.g., in California in 2009, 88% of motorcycle accident fatalities were helmeted, 12% were unhelmeted).*

**SR.3.a** Make lists, tables, Venn diagrams, and tree diagrams to represent all possible outcomes in a sample space of a chance experiment to compute the probability of an event and its complement; interpret their meanings in context. Conduct an experiment or simulation to compute the empirical probability of an event and its complement to draw conclusions or make decisions.

**SR.3.b** Determine whether events are independent or dependent (including through the use of the multiplication rule) and use this information appropriately to determine probabilities.

**SR.3.c** Explain the meaning of conditional probability. Compute conditional and joint probabilities from a given table of data.

*For example: From a two-way frequency table summarizing test outcomes for 500 people, some of whom have cancer and some who do not, determine the probability of having cancer given a positive test result and the probability of having a positive test result given cancer. Choose the probability that is the most informative for a given purpose.*

**SR.4** Recognize the purposes of different types of statistical studies and how randomization relates to each. Use statistics to make informed, evidence-based inferences and justify conclusions from each type of study.

*For example: Given a scenario, determine if it is a survey, experiment, or observational study and what method of randomization (if any) was used. Given a question, determine which data collection method would be the most appropriate way to collect data to answer the question.*

**SR.5** Evaluate reports based on data for appropriateness of the study design, analysis methods, and statistical measures used.

## Algebraic Operations and Functional Analysis

Students should investigate problems that facilitate the transition from specific and numeric reasoning to general and algebraic reasoning. They should use the language, symbols, and structure of algebra and the key characteristics of functions and their representations (symbols, graphs, tables) to investigate, represent, and solve those problems.

Students will:

**AF.1** Use variables to write an algebraic expression to represent a quantity in a problem and interpret expressions that represent a quantity in terms of its context.

> *For example: Be able to use variables in context and use variables as placeholders, as in formulas such as the compound interest formula,* $A = P(1 + r/n)^{nt}$. *Write a spreadsheet formula to calculate prices based on percentage mark-up. Write the expression* $4s + 4$ *to represent the total number of square-foot tiles needed to completely surround a square pool that is s feet on each side; recognize that s represents the length of a side of the pool, that the factor of 4 represents the 4 sides of the pool, and that the addend of 4 represents the tiles needed for 4 corners of the border.*

**AF.2** Rewrite expressions and equations in equivalent forms to reveal and explain properties of the quantity or relationship represented by the expression or equation.

**AF.2.a** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

**AF.2.b** Use algebraic properties and procedures to combine and transform expressions to solve problems, such as factoring to reveal zeros of a function defined by an expression or completing the square to reveal the maximum or minimum value of a function defined by a quadratic expression.

**AF.3** Understand solving a linear equation or inequality, or a system of linear equations or inequalities, as a process of determining which values from a specified set, if any, make the equation, inequality, or system true. Connect numerical, graphical, and symbolic representations of solutions.

**AF.3.a** Recognize that the solution to a linear equation or inequality in one variable, if it exists, is a value or set of values that makes the equation or inequality true, and that the solution corresponds to a single point or an interval on the real number line.

**AF.3.b** Recognize that the solution to a linear equation or inequality in two variables is the set of ordered pairs that makes the equation or inequality true, and that the solution corresponds to a line or a half plane in the coordinate plane.

**AF.3.c** Recognize that a solution to a system of linear equations in two variables, if it exists, is the ordered pair or set of ordered pairs that satisfies both equations in the system, and that the solution to a system of linear equations in two variables corresponds to the point or points of intersection of their graphs.

**AF.3.d** Recognize that solutions to a system of linear inequalities in two variables correspond to points lying in the intersection of the half-planes that contain the solutions to each inequality in the system.

**AF.4** Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function (linear; quadratic; exponential; simple rational of the form $r(x) = \frac{a}{x}$ , $p(x) = \frac{a}{x^2}$ , and $g(x) = \frac{a}{x-c}$ ; and monomial of the form $p(x) = ax^n$ (where n = 2, 3, 4, or 5) that models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

*For example: For linear situations, describe the rate of change using appropriate units, determine the contextual meaning of the rate of change and of the intercepts, and create an algebraic model to represent the function. For exponential situations, interpret the intercept and connect end behavior to growth or decay. For a quadratic function that models the height of a projectile, interpret the maximum value of the function in context. For a simple rational function that represents the relationship between the speed of a train and the time it takes to complete a trip of constant distance, interpret the asymptotes in the context of the situation. Compare the symmetries and the local and end behavior of monomial functions.*

**AF.5** Use linear and quadratic functions and equations to model and solve problems from a variety of contexts and to make predictions/decisions. Recognize the limitations of the model and identify an appropriate domain or range for which the model might be used to make reasonable predictions. Solve equations through estimation using graphs and tables, inspection for simple cases, or algebraic techniques (including factoring and the quadratic formula), when appropriate.

**AF.6** Use exponential functions and equations to model and solve problems from a variety of contexts and to make predictions/decisions.Recognize the limitations of the model and identify an appropriate domain or range for which the model might be used to make reasonable predictions. Solve equations through estimation using graphs and tables or by inspection for integer values.

# Sample Scope and Sequence

| Outcomes | Unit overview | # of Weeks |
|---|---|---|
| SEAD.1,2<br>NR.1,2<br>PR.1,2<br>AF.1 | **Numerical and proportional reasoning.** This unit focuses on authentic problems in a variety of contexts that require number sense and the ability to apply concepts of numeracy to investigate and describe quantitative relationships. Students demonstrate an understanding of rational numbers (including large and small numbers) by interpreting, communicating with, and comparing different forms. They use quantitative information to explore the impact of policies or behaviors on a population, and identify erroneous, misleading, or conflicting information in advertising or consumer information or regarding social, economic, and environmental issues. | 4 |
| SEAD.3<br>SR.1,3,4,5<br>NR.1,2<br>PR.1,2<br>AF.1 | **Statistical and probabilistic reasoning.** This unit addresses data analysis, proportional reasoning, application of percentages in multi-step problems, and probabilistic reasoning. Students read, interpret, and make decisions about data summarized numerically and in graphical displays, such as line graphs, bar graphs, scatterplots, histograms, and Venn diagrams. They learn to interpret statements about chance, risk, and probability that appear in everyday media. | 6 |
| SEAD.4<br>NR.1,2<br>PR.2<br>AF.1,2 | **Algebraic representations and measurement.** This unit addresses geometric reasoning, the use of variables in formulas, and rewriting of algebraic formulas to highlight variables of interest. Students analyze and solve real-world problems involving proportional relationships, including indirect measurement, with attention to appropriate use of units. | 4 |
| PR.2<br>SR.2<br>AF.1,2,3,4,5 | **Linear functions, equations, and inequalities.** This unit focuses on linear relationships, equations, and inequalities. Students determine whether a proportional relationship exists between two quantities, based on how the change in one quantity influences the other quantity. They determine and interpret rates of change and construct linear models to represent situations. Students examine trend lines for approximately linear data. | 5 |
| AF.1,2,3,4,5,6 | **Modeling with linear and exponential functions.** This unit addresses linear and exponential function models. Students move from exploring relationships to formalizing work with functions and covariation. They compare linear relationships to non-linear relationships. Students solve equations that arise from the function models. | 5 |
| AF.1,2,4,5 | **Other nonlinear models.** This unit addresses additional nonlinear models, specifically quadratic, simple monomial, and rational functions. Students use these functions to model and answer questions about a variety of real-world situations and solve equations arising from problem situations. | 5 |

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